LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **CHEMISTRY**

THIRD SEMESTER – NOVEMBER 2007

MT 3103/3101 - MATHEMATICS FOR CHEMISTRY

AB 4

Date : 02/11/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

SECTION A

Answer **ALL** *questions:* $10 \times 2 = 20$

1. Differentiate $\log \sec^{-1}(x^4)$.
2. Find the slope of the curve $y = \dfrac{6x}{x^2 - 1}$ at (2, 4).
3. Evaluate $\int \dfrac{dx}{\sqrt{x(3-2x)}}$.
4. Evaluate $\int \dfrac{dx}{(1+e^x)(1+e^{-x})}$.
5. Prove that $\dfrac{e+1}{e-1} = \dfrac{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots\ldots\ldots}{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\ldots\ldots\ldots}$.
6. Obtain a partial differential equation by eliminating *a*, *b* from $(x-a)^2 + (y-b)^2 + z^2 = 1$.
7. If $\cos(A + iB) = \cos\theta + i\sin\theta$, prove that $\cos 2A + \cosh 2B = 2$.
8. Obtain the Fourier coefficient a_0 for the function $f(x) = x^2$ in $-\pi \le x \le \pi$.
9. State the axioms of probability.
10. Mean and variance of a binomial distribution are 4 and $\dfrac{4}{3}$. Find $P(X \ge 1)$.

SECTION B

Answer any **FIVE** questions: $5 \times 8 = 40$

11. Find the angle of intersection of the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.
12. Prove that $\int_0^{\frac{\pi}{2}} \log \sin x\, dx = \dfrac{\pi}{2}\log\dfrac{1}{2}$.
13. Solve (a) $z = px + qy + pq$.
    (b) $p + q = \sin x + \sin y$. (5 + 3)
14. Evaluate $\int \dfrac{dx}{(x+1)\sqrt{x^2+x+1}}$.
15. Separate the real and imaginary parts of $\tan^{-1}(x+iy)$.
16. Prove that $\dfrac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$.

17. A and B throw alternatively a pair of balanced dices. A wins if he throws a sum of 6 points before B throws a sum of 7 points, while B wins if he throws a sum of 7 points before A throws a sum of 6 points. If A begins the game, show that its probability of winning is $30/61$.

18. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a percolator coffee 75% of the time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chance of having the claim (i) Accepted (ii) Rejected, when he does not have the ability, he claims.

SECTION C

Answer any **TWO** questions: $2 \times 20 = 40$

19. (a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally.

(b) Find the maximum and minimum value of $2x^3 - 3x^2 - 36x + 10$. (10 + 10)

20. (a) Solve $(D^2 + 3D + 2)y = \sin x + e^{2x}$.

(b) $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$. (12 + 8)

21. (a) Sum to infinity the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \ldots\ldots\ldots\ldots$

(b) Find the eigen values and eigen vectors of $\begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$. (6 + 14)

22. (a) Obtain the Fourier series for the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $(0, 2\pi)$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\infty = \frac{\pi^2}{8}$.

(b) Find the mean and variance of Poisson distribution. (12 + 8)

****